Physics 4A
Chapter 8: Dynamics II – Motion in a Plane

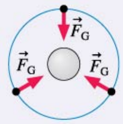
APPLICATIONS

Orbits

An object acted on only by gravity has a circular orbit of radius r if its speed is

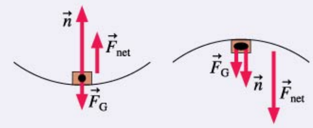
$$v = \sqrt{rg}$$

The object is in free fall.



Circular motion on surfaces

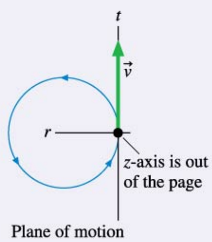
Circular motion requires a net force pointing to the center. n must be > 0 for the object to be in contact with a surface.



IMPORTANT CONCEPTS

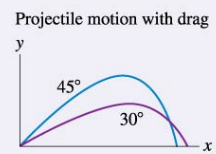
rtz-coordinates

- The r-axis points toward the center of the circle.
- The t-axis is tangent, pointing counterclockwise.



Projectile motion

- With no drag, the x- and y-components of acceleration are independent. The trajectory is a parabola.
- With drag, the trajectory is not a parabola. Maximum range is achieved for an angle less than 45°.



GENERAL PRINCIPLES

Newton's Second Law

Expressed in x- and y-component form:

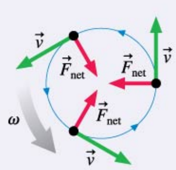
$$(F_{net})_x = \sum F_x = ma_x$$
$$(F_{net})_y = \sum F_y = ma_y$$

Expressed in *rtz*-component form:

$$(F_{net})_r = \sum F_r = ma_r = \frac{mv_t^2}{r} = m\omega^2 r$$
$$(F_{net})_t = \sum F_t = \begin{cases} 0 & \text{uniform circular motion} \\ ma_t & \text{nonuniform circular motion} \end{cases}$$
$$(F_{net})_z = \sum F_z = 0$$

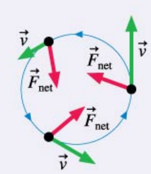
Uniform Circular Motion

- Speed is constant.
- $\vec{F}_{net}$ points toward the center of the circle.
- The **centripetal acceleration** $\vec{a}$ points toward the center of the circle. It changes the particle's direction but not its speed.



Nonuniform Circular Motion

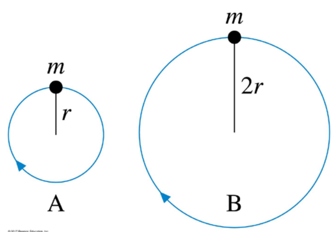
- Speed changes.
- $\vec{F}_{net}$ and $\vec{a}$ have both radial and tangential components.
- The radial component changes the particle's direction.
- The tangential component changes the particle's speed.



Conceptual Questions and Example Problems from Chapter 8

Conceptual Question 8.5

The figure below shows two balls of equal mass moving in vertical circles. Is the tension in string A greater than, less than, or equal to the tension in string B if the balls travel over the top of the circle with equal speed?



8.5. The difference in the tension between A and B is due to the centripetal force $(F_r)_{\text{net}} = \frac{mv^2}{r}$. Since the velocity v is the same for both, the greater radius for B means that the tension in case A is greater than for case B.

Conceptual Question 8.6

Ramon and Sally are observing a toy car speed up as it goes around a circular track. Ramon says, "The car's speeding up, so there must be a net force parallel to the track." "I don't think so," replies Sally. "It's moving in a circle, and that requires centripetal acceleration. The net force has to point to the center of the circle." Do you agree with Ramon, Sally, or neither? Explain.

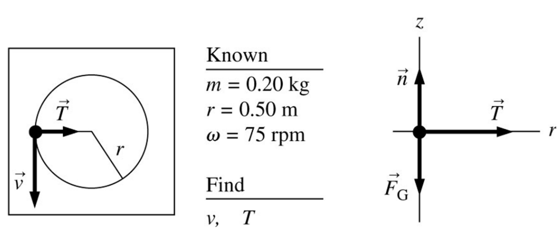
8.6. Neither Sally nor Raymond is completely correct. Both have partially correct descriptions, but are missing key points. In order to speed up, there must be a nonzero acceleration parallel to the track. In order to move in a circle, there must be a nonzero centripetal acceleration. Since both of these are required, the net force points somewhere between the forward direction (parallel to the track) and the center of the circle.

Problem 8.7

A 200 g block on a 50-cm-long string swings in a circle on a horizontal, frictionless table at 75 rpm. (a) What is the speed of the block? (b) What is the tension in the string?

8.7. Model: Treat the block as a particle attached to a massless string that is swinging in a circle on a frictionless table.
Visualize:

Pictorial representation



Solve: (a) The angular velocity and speed are

$$\omega = 75\frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 471.2 \text{ rad/min} \quad v_t = r\omega = (0.50 \text{ m})(471.2 \text{ rad/min}) \times \frac{1 \text{ min}}{60 \text{ s}} = 3.93 \text{ m/s}$$

The tangential velocity is 3.9 m/s.
(b) The radial component of Newton's second law is

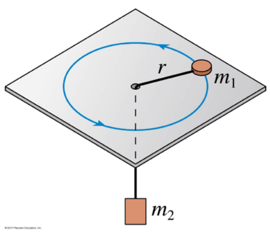
$$\sum F_r = T = \frac{mv^2}{r}$$

Thus

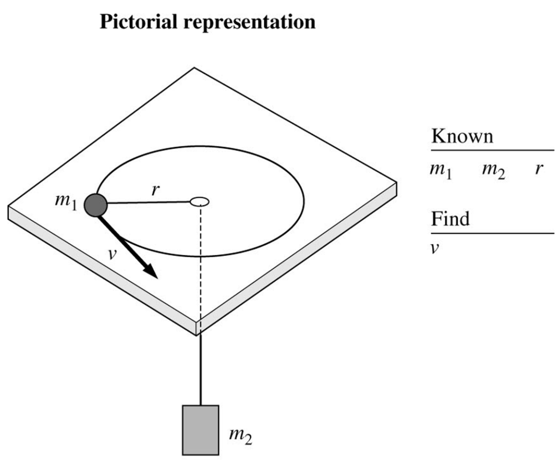
$$T = (0.20 \text{ kg})\frac{(3.93 \text{ m/s})^2}{0.50 \text{ m}} = 6.2 \text{ N}$$

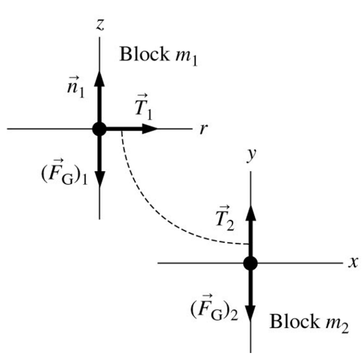
Problem 8.13

Mass m_1 on the frictionless table of the figure to the right is connected by a string through a hole in the table to a hanging mass m_2. With what speed must m_1 rotate in a circle of radius r if m_2 is to remain hanging at rest?



8.13. Model: Masses m_1 and m_2 are considered particles. The string is assumed to be massless.
Visualize:





Solve: The tension in the string causes the centripetal acceleration of the circular motion. If the hole is smooth, it acts like a pulley. Thus tension forces $\vec{T}_1$ and $\vec{T}_2$ act as if they were an action/reaction pair. Mass m_1 is in circular motion of radius r, so Newton's second law for m_1 is

$$\sum F_r = T_1 = \frac{m_1 v^2}{r}$$

Mass m_2 is at rest, so the y-equation of Newton's second law is

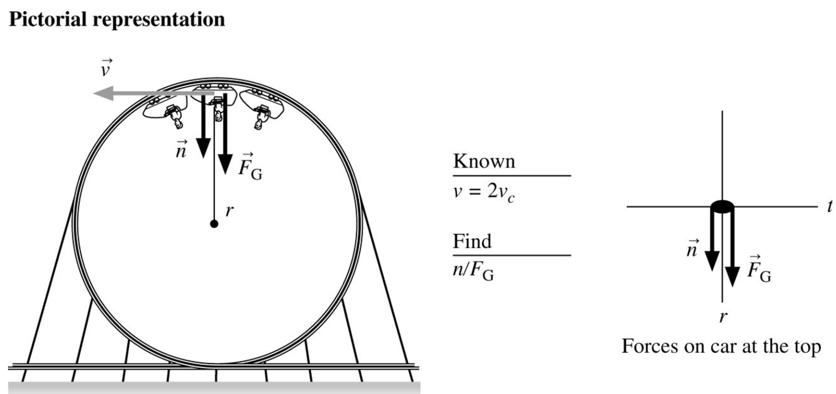
$$\sum F_y = T_2 - m_2 g = 0\ \text{N} \Rightarrow T_2 = m_2 g$$

Newton's third law tells us that $T_1 = T_2$. Equating the two expressions for these quantities:

Problem 8.20

A roller coaster car crosses the top of a circular loop-the-loop at twice the critical speed. What is the ratio of the normal force to the gravitational force?

8.20. Model: Model the roller coaster car as a particle at the top of a circular loop-the-loop undergoing uniform circular motion.
Visualize:



Notice that the r-axis points downward, toward the center of the circle.
Solve: The critical speed occurs when $\vec{n}$ goes to zero and $\vec{F}_G$ provides all the centripetal force pulling the car in the vertical circle. At the critical speed $mg = mv_c^2/r$, therefore $v_c = \sqrt{rg}$. Since the car's speed is twice the critical speed, $v_t = 2v_c$ and the centripetal force is

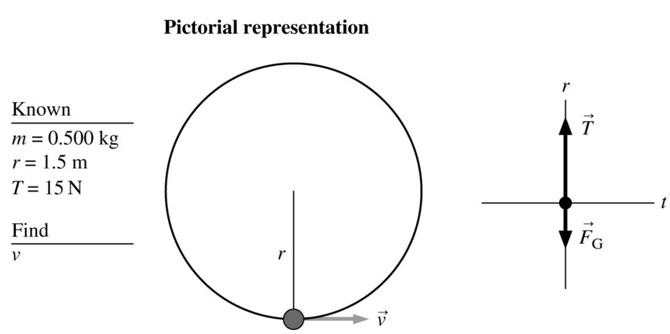
$$\sum F_r = n + F_G = \frac{mv^2}{r} = \frac{m(4v_c^2)}{r} = \frac{m(4rg)}{r} = 4mg$$

Thus the normal force is $n = 3\,mg$. Consequently, $n/F_G = 3$.

Problem 8.24

A 500 g ball swings in a vertical circle at the end of a 1.5-m-long string. When the ball is at the bottom of the circle, the tension in the string is 15 N. What is the speed of the ball at that point?

8.24. Model: Model the ball as a particle which is in a vertical circular motion.
Visualize:



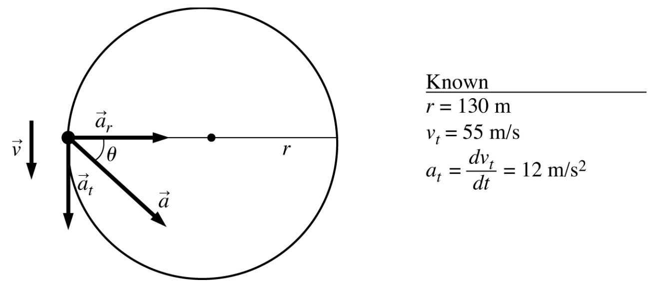
Solve: At the bottom of the circle,

$$\sum F_r = T - F_G = \frac{mv^2}{r} \Rightarrow (15\ \text{N}) - (0.500\ \text{kg})(9.8\ \text{m/s}^2) = \frac{(0.500\ \text{kg})v^2}{(1.5\ \text{m})} \Rightarrow v = 5.5\ \text{m/s}$$

Problem 8.29

An 85,000 kg stunt plane performs a loop-the-loop, flying in a 260-m-diamter vertical circle. At the point where the plane is flying straight down, its speed is 55 m/s and it is speeding up at a rate of 12 m/s per second. **(a)** What is the magnitude of the net force on the plane? You can neglect air resistance. **(b)** What angle does the net force make with the horizontal? Let an angle above the horizontal be positive and an angle below the horizontal be negative.

8.29. Model: The plane is a particle undergoing nonuniform circular motion. Make the *r-t* plane vertical; then there will be on motion in the z-direction. Neglect air resistance.
Visualize:



Solve: We are given the tangential acceleration and we find the centripetal acceleration from $a_r = v^2/r$. Use the second law to find the components of the force.

$$F_r = m\frac{v^2}{r} = (85000\text{ kg})\frac{(55\text{ m/s})^2}{130\text{ m}} = 1.978\times10^6\text{ N}$$

$$F_t = ma_t = (85000\text{ kg})(12\text{ m/s}^2) = 1.02\times10^6\text{ N}$$

(a) Now find the magnitude of the net force.

$$F_{\text{net}} = \sqrt{F_r^2 + F_t^2} = \sqrt{(1.978\times10^6\text{ N})^2 + (1.02\times10^6\text{ N})^2} = 2.2\times10^6\text{ N}$$

(b) Find the angle the net force makes with the horizontal.

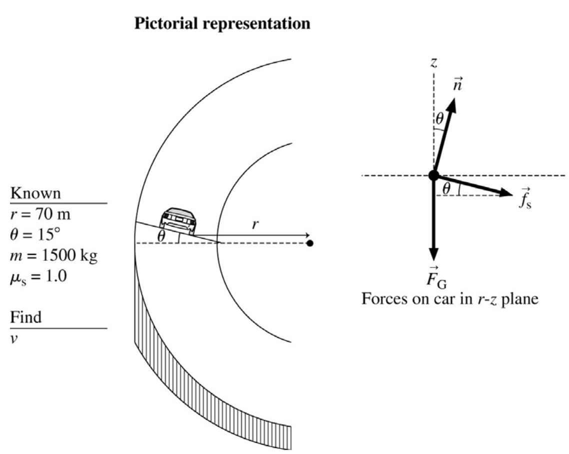
$$\theta = \tan^{-1}\frac{F_t}{F_r} = \tan^{-1}\frac{-1.02\times10^6\text{ N}}{1.978\times10^6\text{ N}} = -27°$$

The angle is negative because it is below the horizontal.
Assess: The plane's velocity is down and increasing, so the net force must be below the horizontal.

Problem 8.40

A concrete highway curve of radius 70 m is banked at a 15° angle. What is the maximum speed with which a 1500 kg rubber-tired car can take this curve without sliding? (μ_s = 1.0)

8.40. Model: We will use the particle model for the car, which is undergoing uniform circular motion on a banked highway, and the model of static friction.
Visualize:



Note that we need to use the coefficient of static friction μ_s, which is 1.0 for rubber on concrete.

Solve: Newton's second law for the car is

$$\sum F_r = f_s \cos\theta + n\sin\theta = \frac{mv^2}{r} \quad \sum F_z = n\cos\theta - f_s \sin\theta - F_G = 0 \text{ N}$$

Maximum speed is when the static friction force reaches its maximum value $(f_s)_{max} = \mu_s n$. Then

$$n(\mu_s \cos 15° + \sin 15°) = \frac{mv^2}{r} \quad n(\cos 15° - \mu_s \sin 15°) = mg$$

Dividing these two equations and simplifying, we get

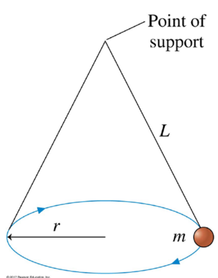
$$\frac{\mu_s + \tan 15°}{1 - \mu_s \tan 15°} = \frac{v^2}{gr} \Rightarrow v = \sqrt{gr\frac{\mu_s + \tan 15°}{1 - \mu_s \tan 15°}}$$

$$= \sqrt{(9.80 \text{ m/s}^2)(70 \text{ m})\frac{(1.0 + 0.268)}{(1 - 0.268)}} = 34 \text{ m/s}$$

Assess: The above value of $34 \text{ m/s} \approx 70 \text{ mph}$ is reasonable.

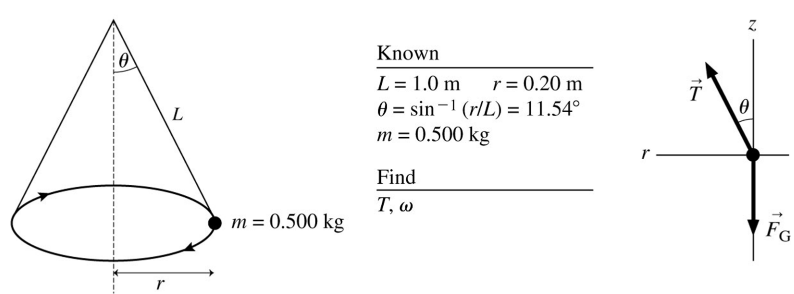
Problem 8.47

A conical pendulum is formed by attaching a ball of mass *m* to a string of length L, then allowing the ball to move in a horizontal circle of radius *r*. The figure below shows that the string traces out the surface of a cone, hence the name. Find an expression for the tension T in the string.



8.47. Use the particle model for the ball, which is undergoing uniform circular motion.
Visualize: We are given L, r, and m, so our answers must be in terms of those variables. L is the hypotenuse of the right triangle. The ball moves in a *horizontal* circle of radius $r = L\cos\theta$. The acceleration and net force point toward the center of the circle, not along the string.

Pictorial representation



Solve:
(a) Apply Newton's second law in the z-direction.

$$\Sigma F_z = T\cos\theta - mg = 0 \Rightarrow T = \frac{mg}{\cos\theta}$$

From the right triangle $\cos\theta = \sqrt{L^2 - r^2}/L$.

$$T = \frac{mg}{\cos\theta} = \frac{mgL}{\sqrt{L^2 - r^2}}$$

(b) Apply Newton's second law in the r-direction.

$$\Sigma F_r = T\sin\theta = m\omega^2 r = m\omega^2(L\sin\theta) \Rightarrow T = m\omega^2 L.$$

Set the two expressions for T equal to each other, cancel m and one L, and solve for ω.

$$\frac{mgL}{\sqrt{L^2 - r^2}} = m\omega^2 L \Rightarrow \omega = \sqrt{\frac{g}{\sqrt{L^2 - r^2}}}$$

(c) Insert $L = 1.0\text{ m}$, $r = 0.20\text{ m}$ and $m = 0.50\text{ kg}$.

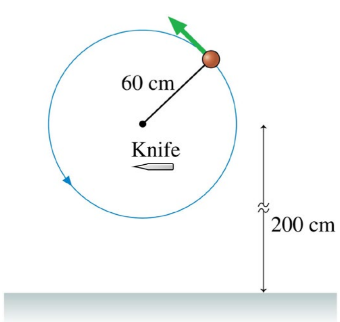
$$T = \frac{mgL}{\sqrt{L^2 - r^2}} = \frac{(0.50\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m})}{\sqrt{(1.0\text{ m})^2 - (0.20\text{ m})^2}} = 5.0\text{ N}$$

$$\omega = \sqrt{\frac{g}{\sqrt{L^2 - r^2}}} = \sqrt{\frac{9.8\text{ m/s}^2}{\sqrt{(1.0\text{ m})^2 - (0.20\text{ m})^2}}} = 3.163\text{ rad/s}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 30\text{ rpm}$$

Assess: Notice that the mass canceled out of the equation for ω, but not for T, so the 500 g was necessary information.

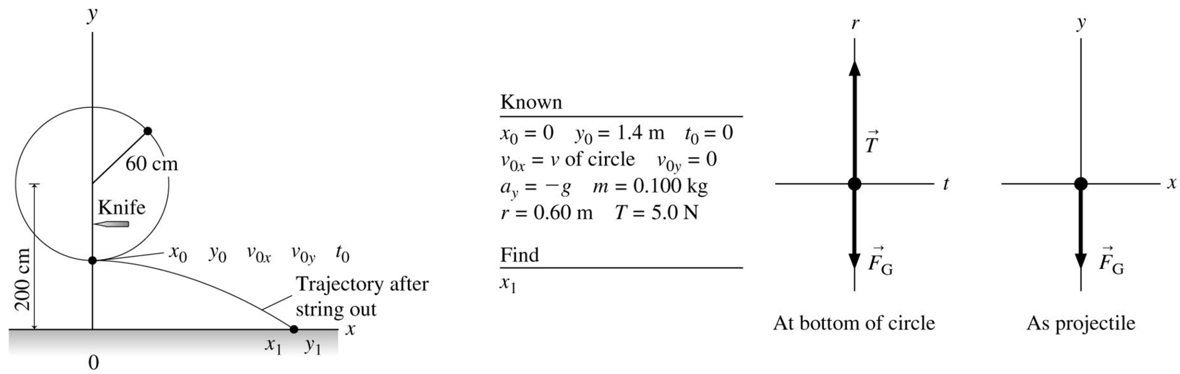
Problem 8.56

A 100 g ball on a 60-cm-long string is swung in a vertical circle about a point 200 cm above the floor. The tension in the string when the ball is at the very bottom of the circle is 5.0 N. A very sharp knife is suddenly inserted, as shown in the figure below, to cut the string directly below the point of support. How far to the right of where the string was cut does the ball hit the floor?



8.56. Model: Model the ball as a particle swinging in a vertical circle, then as a projectile.
Visualize:

Pictorial representation



Solve: Initially, the ball is moving in a circle. Once the string is cut, it becomes a projectile. The final circular-motion velocity is the initial velocity for the projectile. The free-body diagram for circular motion is shown at the bottom of the circle. Since $T > F_G$, there is a net force toward the center of the circle that causes the centripetal acceleration. The r-equation of Newton's second law is

$$(F_{net})_r = T - F_G = T - mg = \frac{mv^2}{r}$$

$$\Rightarrow v_{bottom} = \sqrt{\frac{r}{m}(T - mg)} = \sqrt{\frac{0.60\text{ m}}{0.100\text{ kg}}[5.0\text{ N} - (0.10\text{ kg})(9.8\text{ m/s}^2)]} = 4.91\text{ m/s}$$

As a projectile the ball starts at $y_0 = 1.4$ m with $\vec{v}_0 = 4.91\hat{i}$ m/s. The equation for the y-motion is

$$y_1 = 0\text{ m} = y_0 + v_{0y}\Delta t - \tfrac{1}{2}g(\Delta t)^2 = y_0 - \tfrac{1}{2}gt_1^2$$

This is easily solved to find that the ball hits the ground at time

$$t_1 = \sqrt{\frac{2y_0}{g}} = 0.535\text{ s}$$

During this time interval it travels a horizontal distance

$$x_1 = x_0 + v_{0x}t_1 = (4.91\text{ m/s})(0.535\text{ s}) = 2.63\text{ m}$$

So the ball hits the floor 2.6 m to the right of the point where the string was cut.

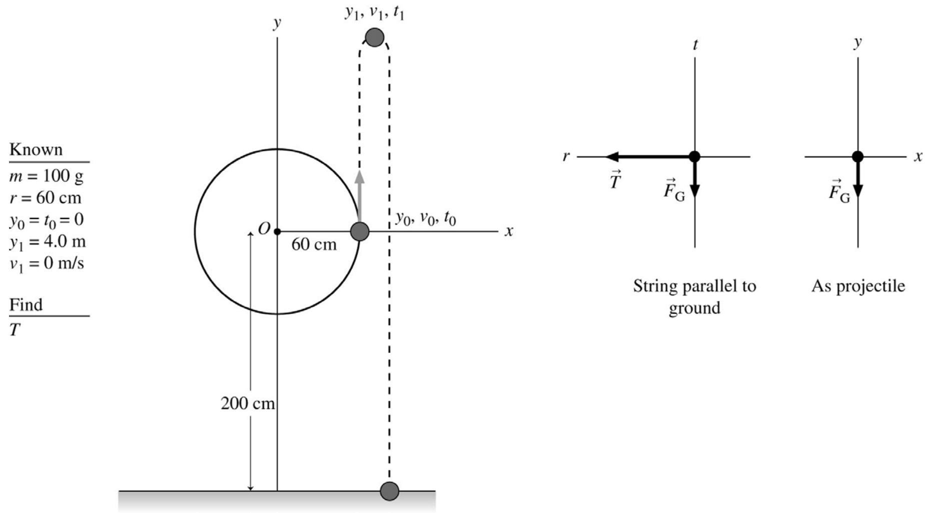
Problem 8.59

A 100 g ball on a 60-cm-long string is swung in a vertical circle about a point 200 cm above the floor. The string suddenly breaks when it is parallel to the ground and the ball is moving upward. The ball reaches a height of 600 cm above the floor. What was the tension in the string an instant before it broke?

8.59. Model: Model the ball as a particle undergoing circular motion in a vertical circle.

Visualize:

Pictorial representation



Solve: Initially, the ball is moving in circular motion. Once the string breaks, it becomes a projectile. The final circular-motion velocity is the initial velocity for the projectile, which we can find by using the kinematic equation

$$v_1^2 = v_0^2 + 2a_y(y_1 - y_0) \Rightarrow 0\ \text{m}^2/\text{s}^2 = (v_0)^2 + 2(-9.8\ \text{m/s}^2)(4.0\ \text{m} - 0\ \text{m}) \Rightarrow v_0 = 8.85\ \text{m/s}$$

This is the speed of the ball as the string broke. The tension in the string at that instant can be found by using the r-component of the net force on the ball:

$$\sum F_r = T = m\left(\frac{v_{0y}^2}{r}\right) \Rightarrow T = (0.100\ \text{kg})\frac{(8.85\ \text{m/s})^2}{0.60\ \text{m}} = 13\ \text{N}$$